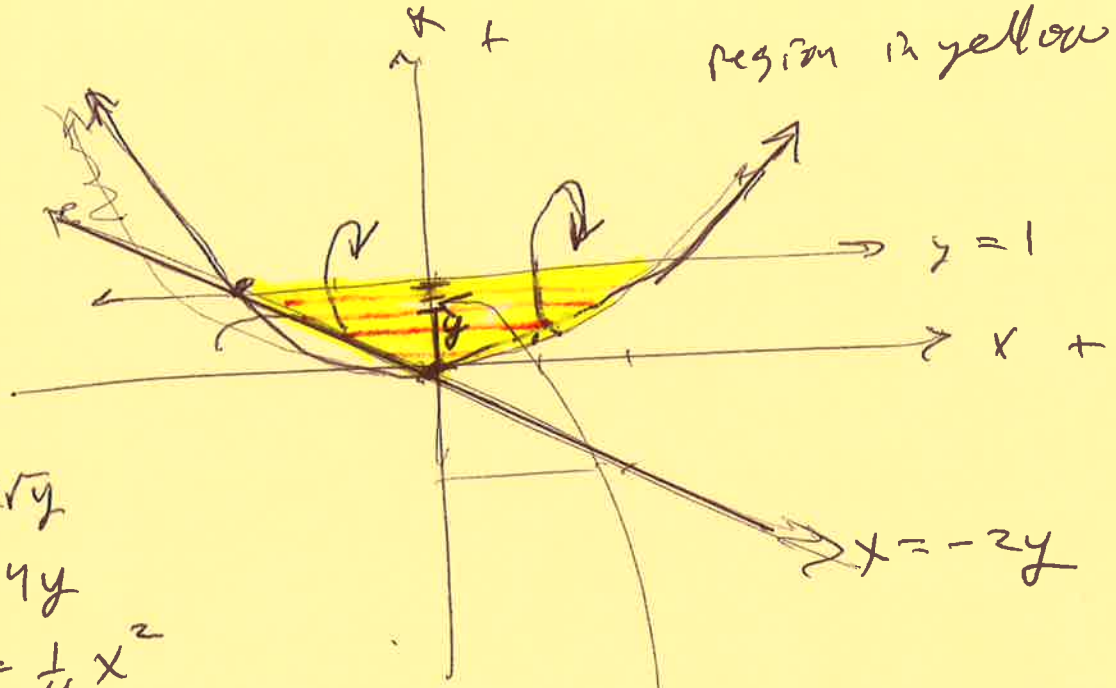
Volume of region? Lect. Review Problems

#9. $x = 2\sqrt{y}$, $x = -2y$ And $y = 1$

About line $y = 1$



$x = 2\sqrt{y}$

$x^2 = 4y$

$y = \frac{1}{4}x^2$

By shells (in red)

radius is $1-y$

$x = -2y$

$y = -\frac{1}{2}x$

$$V = 2\pi \int_0^1 (1-y)\left(2\sqrt{y} - (-2y)\right)\, dy$$

Always go in pos. direction of y. If going from $y=1$ on this one $\int_1^0$ it is in opp. direction $-dy$ which still makes it $\int_0^1$

$$V = 2\pi \int_0^1 2\sqrt{y} + 2y - 2y^{3/2} - 2y^2\, dy$$

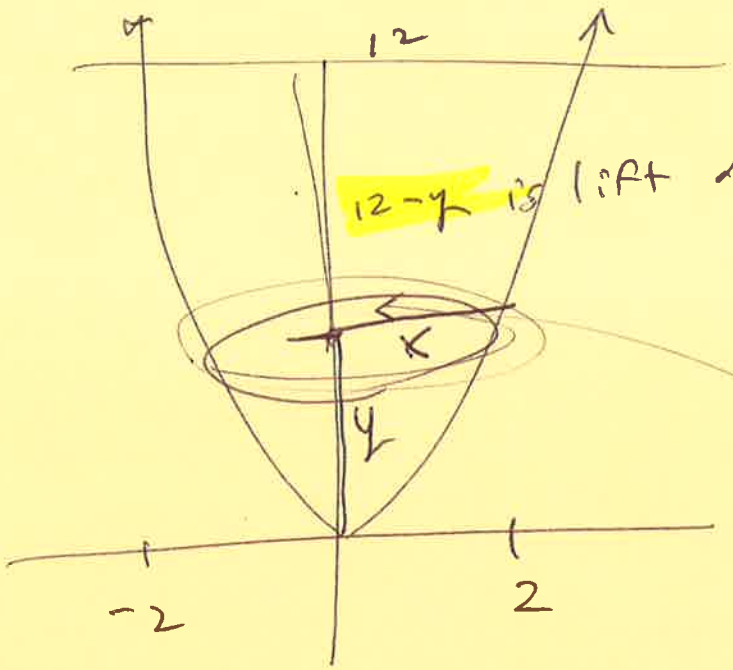
$$= 2\pi\left(\frac{2}{3}2y^{3/2} + \frac{2y^2}{2} - 2y^{5/2}\cdot\frac{2}{5} - \frac{2y^3}{3}\Big|_0^1\right) = 2\pi\left(\frac{13}{15}\right)$$

$$2\pi\left(4/3 + 1 - 4/5 - 2/3\right) = 2\pi\left(\frac{5}{3} - \frac{4}{5}\right) = 2\pi\left(\frac{25}{15} - \frac{12}{15}\right) = \frac{26\pi}{15}$$

Test 2 Lecture Review Problems

Burger
M76

#11



concept.

$$W = \int F(x)\,dx$$

Lifting $W = \int_a^b \rho g\, A(y)\,(h-y)\,dy$

$y = 3x^2 \longrightarrow \sqrt{\frac{y}{3}} = x$ (take pos. $\sqrt{}$)

$A(y) = \pi x^2 = \pi\left(\sqrt{\frac{y}{3}}\right)^2 = \frac{\pi y}{3}$

$\rho g = 9800\ N/m^3$

$$W = \int_0^{12} 9800\left(\frac{\pi y}{3}\right)(12-y)\,dy$$

$$= \frac{9800\pi}{3}\int_0^{12} 12y - y^2\,dy = \frac{9800\pi}{3}\left[\frac{12y^2}{2} - \frac{y^3}{3}\,\Big|_0^{12}\right]$$

$$\frac{9800\pi}{3}\left(6(144) - \frac{(12)^3}{3}\right) = 21{,}955{,}610\ J$$

$6y^2 - \frac{y^3}{3}$